Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 51326

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Third/Fourth Semester

Biomedical Engineering

MA 3355 — RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to : Electronics and Communication Engineering/Electronics and Telecommunication Engineering/Medical Electronics)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

(Statistical tables may be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the axioms of probability.

2. The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of this distribution.

3. State central limit theorem.

4. If X and Y are independent random variables prove that $\text{Cov}(X, Y) = 0$.

5. State the four classifications of Random processes.

6. State Chapman Kolmogorov equations.

7. Is the set of all matrices A such that $\det(A) = 0$ subspaces of the matrix M_{mn}? Justify.

8. Why $v_1 = (-1, 2, 4)$ and $v_2 = (5, -10, -20)$ in R^3 are linearly dependent? Explain it.

9. What is Identity transformation?

10. Write eigen values of the matrices A^2 and A^{-1} given that matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The CDF of the random variable X is defined by

$$F_X(x) = \begin{cases} 0, & x < 2 \\ C(x-2), & 2 \le x < 6 \\ 1, & x \ge 6 \end{cases}$$

(1) What is the value of C? (8)

(2) With the above value of C, what is $P[X > 4]$?

(3) With the above value of C, what is $P[3 \le X \le 5]$?

(ii) The average percentage of marks of candidates in an examination is 42 will a standard deviation of 10, the minimum for a pass is 50%. If 1000 candidates appear for the examination, how much can be expected marks. If it is required, that double that number should pass, what should be the average percentage of marks? (8)

Or

(b) (i) A shopping cart contains ten books whose weights are as follows:

There are four books with a weight of 1.8 lbs each, one book with a weight of 2 lbs, two books with a weight of 2.5 lbs each, and three books with a weight of 3.2 lbs each.

(1) What is the mean weight of the books?

(2) What is the variance of the weights of the books? (8)

(ii) Given that X is normally distribution with mean 10 and probability $P(X > 12) = 0.1587$. What is the probability that X will fall in the interval $(9, 11)$. (8)

12. (a) The joint PDF of the random variables X and Y is defined as follows :

$$f_{XY}(x, y) = \begin{cases} 25e^{-5y}, & 0 < x < 0.2,\ y > 0 \\ 0 & , \text{otherwise} \end{cases}$$

(i) Find the marginal PDFs of X and Y

(ii) What is the correlation coefficient of X and Y? (16)

Or

(b) (i) The details pertaining to the experience of technicians in a company (in a number of years) and their performance rating is provided in the table below. Using these values, fit the straight line. Also estimate the performance rating for a technician with 20 years of experience. (10)

Experience of Technicians (in year)	16	12	18	4	3	10	5	12
Performance rating	87	88	89	68	78	80	75	83

(ii) The joint PDF of two random variables X and Y is given by $f_{XY}(x, y)$. If we define the random variable $U = XY$, determine the PDF of U. (6)

13. (a) (i) Suppose we are interested in $X(t)$ but we can observe only $Y(t) = X(t) + N(t)$ where $N(t)$ is a noise process that interferes with our observation of $X(t)$. Assume $X(t)$ and $N(t)$ are independent wide sense stationary processes with $E[X(t)] = \mu_X$ and $E[N(t)] = \mu_N = 0$. Is $Y(t)$ wide sense stationary? (8)

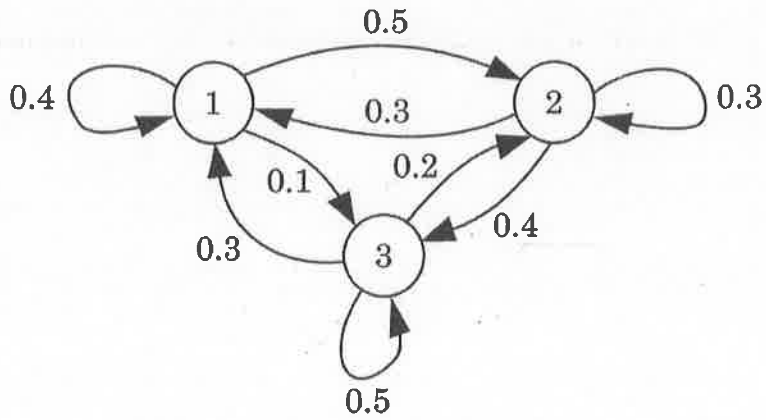
(ii) Cars, trucks, and buses arrive at a toll booth as independent Poisson processes with rates $\lambda_c = 1.2$ cars/minute, $\lambda_t = 0.9$ trucks/minute, and $\lambda_b = 0.7$ buses/minute. In a 10-minute interval, what is the PMF of N, the number of vehicles (cars, trucks, or buses) that arrive? (8)

Or

(b) (i) Check whether the Markov chain with transition probability matrix

$$p = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$$ is irreducible or not? (8)

(ii) Find the transition probability matrix and the limiting-state probabilities of the process represented by the state-transition diagram shown in Figure (8)



14. (a) (i) Check whether the set of all pairs of real numbers of the form $(1, x)$ with the operations $(1, x_1) + (1, x_2) = (1, x_1 + x_2)$ and $k(1, x) = (1, kx)$ is vector space. (8)

(ii) Consider the vectors $u = (1, 2, -1)$ and $v = (6, 4, 2)$ in R^3. Show that $w = (9, 2, 7)$ is a linear combination of u and v. (8)

Or

(b) (i) Show that the vectors $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$, $v_3 = (3, 3, 4)$ form a basis for R^3. (8)

(ii) Find the dimension and a basis for the solution space W of the system of homogeneous equation given below. (8)

$$x_1 + 2x_2 + 2x_3 - x_4 + 3x_5 = 0$$

$$x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 0$$

$$3x_1 + 6x_2 + 8x_3 + x_4 + 5x_5 = 0$$

15. (a) (i) State and prove Dimension theorem. (10)

(ii) Show that the transformation $T : R^3 \to R^2$ defined by $T(x, y, z) = (z, x + y)$ is linear. (6)

Or

(b) (i) Find an orthonormal basis of R^3, given that an arbitrary basis is $\{v_1 = (3, 0, 4), v_2 = (-1, 0, 7), v_3 = (2, 9, 11)\}$. (8)

(ii) Find a basis and dimension of R_T and N_T for the linear transformation $T : R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$. (8)